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The Extremal and Infra-Topological Spaces over a Finite Set-A Computational Approach with Maple.

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Highlights

- **Extremal and infra-topologies on finite sets were constructed listed and enumerated.**
- **Maple code automates their generation, listing, and computation. Results are verified against theoretical expectations.**
- **Efficient enumeration of extremal topologies for larger sets.**

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ABSTRACT

The main purpose of this paper is to create a suitable maple code to generate an extremal topology and to compute and list all extremal topologies over a finite set of size less than or equal to 5 points and calculate their number and the number of open sets in a generated extremal topology. The Codes were developed to check if these numbers are identical to the obtained formulas. Moreover, maple procedures have been created to compute the infra-topological spaces and their number over a finite set. Computations are carried out by using Maple 2022.

1. Introduction

Papzyan (1991) introduced and proved the concept of extremal topologies by using semi-group to define them (Mera & Sola, 2005). The extremal topology is the topology that every topology which strictly finer than it is a discrete topology (Mera & Sola, 2005). After a few years, Mera & Sola (2005) presented a beautiful mathematical work that proved the existence theorem for extremal topologies 1-2 theorem (Mera & Sola, 2005), and they provided a new theorem that helps to show that each extremal topology on a finite set should take certain form 2-1 theorem (Mera & Soila, 2005). In addition, they utilized the two theorems to prove a counting theorem, which indicates the number of extremal topologies on a n-element set 2-2 theorem (Mera & Sola, 2005). In addition, they provide a theorem that counts the number of open sets of an extremal topology 2-3 theorem (Mera & Sola, 2005). The aim of this paper is to create maple procedures that generate an extremal topology over a finite set and to compute and list all extremal topologies over the given set. Also, there are procedures that compute the number of open sets in the generated extremal topology and verify if these obtained numbers are identical to the formulas in the indicated paper. Moreover, there is a procedure created to find all infra-topological spaces over a finite set, which is a topology such that the only topology strictly coarser than it is the trivial topology (Larson & Andima, 1995). Every in-Fra space topology over X has the form {∅, A, X} and A is a proper subset of X (Larson & Andima, 1995). In addition, to compute their numbers over a finite set X.

Note: In this paper, it has been listed all extremal topologies have-over a set of sizes less than or equal to 5 points, while the procedure can list the result for a set with 11 points size

2. Preliminaries

2-1. Theorem

If X is any set with more than one element, x , y ∈ X , x ≠ y, and τ{x , y}=P(X\{x})∪{{x}∪A :A∈ P(X\{x}), y∈A }, then τ{x ,y} is an extremal topology on X (Mera & Sola, 2005).

2-2 Theorem

If X is a set with n elements, then the number of extremal topologies defined on X is n(n−1), (Mera & Sola, 2005).

2-3 Theorem

If X is a set with n elements, then any extremal topology has $3(2^{n-2})$ elements, (Mera & Sola, 2005).

3.3. Computations

```
>with(combinat):
>#(1) A procedure to represent the finite set X;  FS:=proc(n) local i;{seq(x[i],i=1..n)};  end:#(2)A procedure to generate extremal topology over X by x,y;  extremal:=proc(x,y,X)
local o,c,t,e,px,py,M,txy;
```

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```
txy:=e union {x};
M:=M union {txy};
od;
px union M;
else flase;
fi;
end:
#(3) A procedure to find all extremal topologies
over a finite set X;
ALLExtremals:=proc(X)
local EX,x,y;
EX:={};
for x in X do
for y in X minus {x} do
EX:=EX union {extremal(x, y, X)};
od;
od;
EX;
end:
#(3) A procedure to find all extremal topologies
 over a finite set X;
ALLExtremals:=proc(X)
local EX,x,y;
EX:={};
for x in X do
for y in X minus {x} do
EX:=EX union {extremal(x, y, X)};
od;
od;
EX;
end:
#(4) A procedure to find the Infra-Topological spaces
 over a finite set;
Infra_Topology:=proc(X)
local o,c,PX,T;
c:={};T:={};
PX:=powerset(X);
for o in PX do
c:=c union{{{}, o, X}};
od;
c minus {{{}, X}};
c:={};M:={};txy:={};
px:=powerset(X minus {x});
for o in px do
if `member`(x,X) and `member`(y, X) and x<>y then
>#Example(1);
n:=2;
n:=2
>X:=FS(n);
X:={x1,x2}
>Generated_Extremal_Topology:=extremal(x[1],x[2],X);
```

Generated_Extremal_Topology :={ Ø, {x_2}, {x_1, x_2}}

>print (`Numbers Of Open Sets is`, nops(Generated_Extremal_Topology));

Numbers Of Open Sets is, 3

>print(`Check if The Number of Open Sets is Identical with The Formula in Theorem 2-3 of (Mera & Sola, 2005)`, evalb(nops(Generated_Extremal_Topology)=3*(2^(n-2))));

Check if The Number of Open Sets is Identical to The Formula in Theorem 2-3 of (Mera & Sola, 2005), true

> EXTREMALS:=ALLExtremals (X);

EXTREMALS = {{Ø, {x_1}, {x_1, x_2}}, {Ø, {x_2}, {x_1, x_2}}}

>print("Number of Extremal Topologies is `, nops (EXTREMALS), over a set with`, n,`points`);

Number of Extremal Topologies is, 2, over a set with, 2, points

>print("Check if The Number of Extremal Topologies is Identical with The Formula in Theorem 2-2 of (Mera & Sola, 2005)`,evalb (nops (EXTREMALS)= n* (n-1)));

Check if The Number of Extremal Topologies is Identical to The Formula in Theorem 2-2 of (Mera & Sola, 2005), true

> infra topological_spaces :=Infra_Topology (X);

infra_topological_spaces = {{Ø, {x1}, {x_1, x_2}}, {Ø, {x_2}, {x_1, x_2}}}

> print("The number of infra-topological spaces is`,nops(infra_topological_spaces), `over a set with`, nops (X), `points`,evalb (nops (infra topological_spaces)=(2^n)-2));

The number of infra-topological spaces is, 2, over a set with, 2, points, true

>#Exampl (2);

>n:=3;

n:=3

>X:=FS(n);

X:={x_1, x_2, x_3}

>Generated_Extremal_Topology:=extremal(x[2],x[3],X);

Generated_Extremal_Topology := { Ø,{x_1},{x_3},{x_1, x_3},{x_2, x_3},{x_1, x_2, x_3}}

>print(`Numbers Of Open Sets is`, nops(Generated_Extremal_Topology));

Number of Open Sets is 6

>print (Check if The Number of Open Sets is Ident cal with The Formula in Theorem 2-3 of (Mera & Sola 2005), evalb(nops(Gener-ated_Extremal_Topology)=3*(2^(n-2))));

Check if The Number of Open Sets is Identical to The Formula in Theorem 2-3 (Mera & Sola, 2005).

>EXTREMALS:=ALLExtremals(X);

EXTREMALS={{ Ø, {x_1}, {x_2}, {x_1, x_2}, {x_1, x_3}, {x_1, x_2, x_3}}, { Ø, {x_1}, {x_2}, {x_1, x_2}, {x_2, x_3},{x_1, x_2, x_3}}, { Ø, {x_1}, {x_3}, {x_1, x_2}, {x_1, x_3}, {x_1, x_2, x_3}}, { Ø, {x_1}, {x_3}, {x_1, x_3}, {x_2, x_3}{x_1, x_2, x_3}}, { Ø, {x_2}, {x_3}, {x_1, x_2}, {x_2, x_3}, {x_1, x_2, x_3}}, { Ø, {x_2}, {x_3}, {x_1, x_3}, {x_2, x_3}, {x_1, x_2, x_3}}};

>print(`Number of Extremal Topologies is `,nops(EXTREMALS),`over a set with`,n,`points`); Number of Extremal Topologies is, 6, over a set with, 3, points

>print(`Check if The Number of Extremal Topologies is Iden-tical with The Formula in Theorem 2-2 of (Mera & Sola, 2005)`, evalb(nops(EX-TREMALS)= n*(n- 1)));

Check if The Number of Extremal Topologies is Identical with The Formula in Theorem 2 -2 of (Mera & Sola, 2005), true

>infra_topological_spaces :=Infra_Topology(X);

infra_topological_spaces = {{Ø, {x_1}, {x_1, x_2, x_3}}, { Ø, {x_2}, {x_1, x_2, x_3}}, { Ø, {x_3}, {x_1, x_2,x_3}}, {Ø, {x_1, x_2}, {x_1, x_2, x_3} }, { Ø, {x_1, x_3}, {x_1, x_2, x_3}}, { Ø, {x_2, x_3}, {x_1, x_2, x_3}}}

>print(`The number of infra-topological spaces is`, nops(infra_topological_spaces),`over a set with`, nops(X), `points`, evalb (nops (infra_topological_spaces)=(2^n)-2)); The number of infra-topological spaces is, 6, over a set with, 3, points, true

>#Example (3);

> n:=4;

n:= 4

>X:=FS(n);

X:={x_1, x_2, x_3, x_4}

>Generated_Extremal_Topology:=extremal(x[2],x[3],X);

Generated_Extremal_Topology:={Ø,{x_1},{x_3}, {x_4}, {x_1, x_3}, {x_1, x_4}, {x_2, x_3}, {x_1, x_2, x_3}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}

>print(`Numbers Of Open Sets is`,nops(Generated_Extremal_Topology));

Numbers Of Open Sets is, 12

>print(`Check if The Number of Open Sets is Identical with The Formula in Theorem 2-3 of (Mera & Sola 2005), evalb (nops (Gener-ated_Extremal_Topology)=3*(2^(n-2))));

Check if The Number of Open Sets is Identical to The Formula in Theorem 2-3 (Mera & Sola, 2005).

EXTREMALS:=ALLExtremals(X);

{{Ø,{x_1}, {x_2}, {x_3},{x_1, x_2}, {x_1, x_3},{x_1, x_4}, {x_2, x_3}, {x_1, x_2, x_3}, {x_1, x_2,x_4}, {x_1, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_2}, {x_3}, {x_1, x_2}, {x_1, x_3}, {x_2, x_3}, {x_2, x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_2}, {x_3}, {x_1, x_2}, {x_1, x_3}, {x_2, x_3}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_2}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_2 x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_2}, {x_4}, {x_1,x_2}, {x_1,x_4}, {x_2,x_3}, {x_2,x_4}, {x_1,x_2,x_3}, {x_1,x_2, x_4}, {x_2, x_3,x_4}, {x_1, x_2, x_3,x_4}}, {Ø, {x_1}, {x_2}, {x_4}, {x_1, x_2}, {x_1, x_4}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_3}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_3}, {x_4}, {x_1, x_3}, {x_1, x_4}, {x_2, x_3}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1}, {x_3}, {x_4}, {x_1, x_3}, {x_1, x_4}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_2}, {x_3}, {x_4}, {x_1, x_2}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_2}, {x_3}, {x_4}, {x_1, x_3}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø, {x_2}, {x_3}, {x_4}, {x_1, x_4}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}}

>print(`Number of Extremal Topologies is `,nops(EXTREM-ALS),`over a set with`,n,`points`);

Number of Extremal Topologies is ,12, over a set with, 4, points

>print(`Check if The Number of Extremal Topologies is Iden-tical with The Formula in Theorem 2-2 of (Mera & Sola 2005)`,evalb(nops(EX-TREMALS)= n*(n- 1)));

Check if The Number of Extremal Topologies is Identical with The Formula in Theorem 2-2 of (Mera & Sola 2005) true

>infra_topological_spaces:=Infra_Topology(X); infra_topologi-cal_spaces :={{Ø,{x1}, {x_1,x_2,x_3,x_4}}, {Ø,{x_2}, {x_1,x_2,x_3,x_4}}, {Ø,{x_3}, {x_1,x_2,x_3,x_4}}, {Ø, {x_4}, {x_1, x_2, x_3, x_4}}, {Ø,{x_1, x_2}, {x_1, x_2, x_3, x_4}}, {Ø,{x_1, x_3}, {x_1, x_2, x_3, x_4}}, {Ø, {x_1, x_4}, {x_1, x_2, x_3, x_4}}, {Ø,{x_2, x_3},{x_1, x_2, x_3, x_4}}, {Ø,{x_2, x_4},{x_1, x_2, x_3, x_4}}, {Ø, {x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø,{x_1, x_2, x_3}, {x_1, x_2, x_3, x_4}}, {Ø,{x_1, x_2, x_4}, {x_1, x_2, x_3, x_4}}, {Ø,{x_1, x_3, x_4}, {x_1, x_2, x_3, x_4}}, {Ø,{x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}}}

>print(`The number of infra-topological spaces is`,nops(in-fra_top-ological_spaces),`over a set with`,nops(X),`points`,evalb(nops(in-fra_topologi-cal_spaces)=(2^n)-2));

The number of infra-topological spaces is, 14, over a set with, 4, points, true

>#Example (4);

>n:=5;

n:=5 >

X:=FS(n);

X:={x1,x2,x3,x4,x5}

>Generated_Extremal_Topology:=extremal(x[1],x[2],X); Gener-ated_Extremal_Topology := {Ø,{x_2}, {x_3}, {x_4}, {x_5}, {x_1, x_2}, {x_2, x_3}, {x_2, x_4}, {x_2, x_5}, {x_3, x_4}, {x_3, x_5}, {x_4, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}

>print (`Numbers Of Open Setes is`, nops (Generated_Ex-tremal_To-pology));

Numbers Of Open Sets is, 24

>print(`Check if The Number of Open Sets is Identical with The Formula in Theorem 2-3 of (Mera & Sola, 2005)`,evalb(nops(Gen-erated_Ex-tremal_Topology)=3*(2^(n-2))));

Check if The Number of Open Sets is Identical with The Formula in Theorem 2-3 of (Mera & Sola 2005), true

>EXTREMALS:=ALLExtremals(X);

EXTREMALS := {{Ø, {x_1}, {x_2}, {x_3}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_1, x_5}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_2, x_3}, {x_2, x_4}, {x_2, x_5}, {x_3, x_4}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_3, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_1, x_3, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_4}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_2, x_3}, {x_2, x_4}, {x_3, x_4}, {x_4, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_3, x_4}, {x_1, x_4, x_5}, {x_2, x_3, x_4}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_5}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_1, x_5}, {x_2, x_3}, {x_2, x_5}, {x_3, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_5}, {x_1, x_2}, {x_1, x_3}, {x_1, x_5}, {x_2, x_3}, {x_2, x_4}, {x_2, x_5}, {x_3, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_5}, {x_1, x_2}, {x_1, x_3}, {x_1, x_5}, {x_2, x_3}, {x_2, x_5}, {x_3, x_4}, {x_3, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_1, x_3, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_3}, {x_5}, {x_1, x_2}, {x_1, x_3}, {x_1, x_5}, {x_2, x_3}, {x_2, x_5}, {x_3, x_5}, {x_4, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_5}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_4}, {x_5}, {x_1, x_2}, {x_1, x_3}, {x_1, x_4}, {x_1, x_5}, {x_2, x_4}, {x_2, x_5}, {x_4, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4,x_5}, {x_1, x_3, x_4,x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_4}, {x_5}, {x_1, x_2}, {x_1, x_4}, {x_1, x_5}, {x_2, x_3}, {x_2, x_4}, {x_2, x_5}, {x_4, x_5}, {x_1, x_2, x_3}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_5}, {x_1 x_2, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_4}, {x_5}, {x_1, x_2}, {x_1, x_4}, {x_1, x_5}, {x_2, x_4}, {x_2, x_5}, {x_3, x_4}, {x_4, x_5}, {x_1, x_2, x_4}, {x_1, x_2, x_5}, {x_1, x_3, x_4}, {x_1, x_4, x_5}, {x_2, x_3, x_4}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1}, {x_2}, {x_4}, {x_5}, {x_1, x_2}, {x_1, x_4},{x_1, x_5}, {x_2, x_4}, {x_2, x5}, {x3, x5}, {x4, x5}, {x1, x2, x4}, {x_1, x_2, x_5}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_2}, {x_3}, {x_4}, {x_5}, {x_1, x_5}, {x_2, x_3}, {x_2, x_4}, {x_2, x_5}, {x_3, x_4}, {x_3, x_5}, {x_4, x_5}, {x_1, x_2, x_5}, {x_1, x_3, x_5}, {x_1, x_4, x_5}, {x_2, x_3, x_4}, {x_2, x_3, x_5}, {x_2, x_4, x_5}, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_4, x_5}, {x_1, x_3, x_4, x_5}, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}}

> print("Number of Extremal Topologies is `, nops (EXTREM-ALS), `over a set with`, n,`points`);

Number of Extremal Topologies is, 20, over a set with, 5, points

>print("Check if The Number of Extremal Topologies is Identi-cal with The Formula in Theorem 2-2 of (Mera & Sola, 2005)`,evalb (nops (EXTREMALS)= n*(n-1)));

Check if The Number of Extremal Topologies is Identical to The Formula in Theorem 2-2 of (Mera & Sola, 2005), true

> infra_topological_spaces:=Infra_Topology(X);

Infra_topological_spaces = {{ Ø, {x_1}, {x_1, x_2, x_3, x_4, x_5} }, { Ø, {x_2}, {x_1, x_2, x_3, x_4, x_5}}, { Ø, {x_3}, {x_1, x_2, x_3, x_4, x_5}}, { Ø, {x_4}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_5}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_1, x_2}, {x_1, x_2, x_3, x_4 x_5}}, {Ø, {x_1, x_3}, {x_1, x_2, x_3, x_4 x_5} }, {Ø, {x_1, x_4}, {x_1, x_2, x_3, x_4 x_5}}, {Ø, {x_1, x_5}, {x_1, x_2, x_3, x_4 , x_5} }, {Ø, {x_2, x_3}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_2, x_4}, {x_1, x_2, x_3, x_4 x_5}}, {Ø, {x_2, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_3, x_4}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_3, x_5}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_4,x_5}, {x_1, x_2, x_3, x_4, x_5} },

{Ø, {x_1, x_2, x_3}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_2, x_4}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_2, x_5}, {x_1, x_2, x_3, x_4, x_5}}, { Ø, {x_1, x_3, x_4}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_3, x_5}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_1, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_2, x_3, x_4}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_2, x_3, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_2, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5} }, {Ø, {x_1, x_2, x_3, x_4}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_2, x_3, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_2, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_1, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4, x_5}}, {Ø, {x_2, x_3, x_4, x_5}, {x_1, x_2, x_3, x_4 ,x_5}}}

> print("The number of infra-topological spaces is`,nops (infra_topological_spaces),`over a set with`,nops(X),`points`,evalb (nops (infra topological_spaces)=(2^n)-2));

The number of infra-topological spaces is, 30, over a set with, 5, points, true

4. Conclusion

In this paper, an extremal topology is constructed on a finite set, and the number of open sets within it is determined. Additionally, all possible extremal topologies that can be derived from the given finite set are identified and enumerated. These results are obtained by translating abstract mathematical formulas and theories into Maple code, and they have been verified to be consistent with theoretical expectations. Furthermore, Maple procedures have been developed to systematically generate all possible infra-topologies and compute their count when defined on a finite set. Since manually listing all extremal topologies becomes impractical for sets with more than four elements, the use of these computational procedures facilitates the calculations and provides further validation for the presented theoretical findings.

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