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Similarity-Based Uncertainty Modeling in Liver Disease Diagnosis: A Comparative Analysis of Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Frameworks.

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Highlights

- **Presenting a systematic comparative study of fuzzy, intuitionistic fuzzy and neutrosophic sets, analyzing the properties and algebraic operations of each framework in handling ambiguity and uncertainty.**
- **Applying similarity measures based on the three frameworks to real clinical data of key liver enzymes (AST, ALT, and bilirubin) for seven patients, to assess the degrees of convergence between patient data and reference measurements.**
- **The results indicate that classical sets fail to address ambiguity, whereas fuzzy sets allow partial membership assessment, intuitionistic fuzzy sets distinguish between membership and non-membership, and neutrosophic sets excel through their independent representation of truth, indeterminacy, and falsity to handle incomplete and contradictory information.**
- **The weighted average is presented (or: Presenting the weighted average) to conduct a comprehensive health status assessment for each patient, relying on similarity measures as key tools to clarify the scientific differences among the three theories in quantifying information proximity.**
- **The results underscore the superiority of the neutrosophic similarity measure, as it yields more precise discrimination in borderline or ambiguous cases, thereby enhancing the accuracy of medical decisions relative to other frameworks.**

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ABSTRACT

This study presents a systematic comparison of three frameworks for modeling uncertainty: fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets, and their ability to represent ambiguity and uncertainty. It employs an analytical review of the literature, focusing on the characteristics of each framework and its key algebraic operations.

Similarity measures of the three frameworks are then examined and applied to real clinical data of the most important liver enzymes (AST, ALT, bilirubin) for seven patients. The results indicate that classical sets are unable to deal with ambiguity, while fuzzy sets allow for partial assessment of membership. Intuitionistic fuzzy sets distinguish between membership and non-membership, whereas neutrosophic sets represent three independent components (truth, indeterminacy and falsity), enhancing their ability to assess incomplete and contradictory uncertainty. Applying similarity measures demonstrates how each framework presents the degree of convergence of patient data with each other and with reference measures. The weighted average allows for comprehensive assessment of patient health status. Therefore, similarity measures serve as important elements in clarifying the scientific differences between the three theories, explaining how each framework measures information proximity differently and its ability to represent components. Results indicate that neutrosophic similarity provides the most precise distinction in borderline or ambiguous cases, enhancing medical decision-making accuracy.

1. Introduction

The concept of uncertainty is a fundamental principle across many scientific fields, ranging from philosophy and logic to artificial intelligence and decision-making systems. Uncertainty refers to situations where information is incomplete or inaccurate, making it difficult to reach a definitive conclusion or make a decision with complete confidence. Ambiguity, on the other hand, relates to the existence of multiple interpretations or equal probabilities for a particular event or phenomenon, making it difficult to understand. The importance of studying uncertainty has grown with the development of mathematical and statistical models. Classical set theory is based on the binary principle of membership, where an element either belongs to a set or does not. This concept forms the basis for most other branches of mathematics, from algebra and analysis to logic (Roitman, 1990). This theory originated in the late 19th century with the mathematician Georg Cantor (Purkert and Ilgauds, 2013), who introduced the definition of membership, which is considered one of the fundamental pillars of set theory. In a world where boundaries between concepts blur and degrees of membership vary continuously, the need arose for a mathematical model capable of representing the uncertainty and ambiguity that classical set theory cannot accurately express. This led to the emergence of fuzzy set theory, introduced by the mathematician Lotfi

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Zadeh in 1965 (Zadeh *et al.*, 1996) as a flexible mathematical model allowing for partial membership between 0 and 1, thus reflecting reality more accurately in ambiguous situations. This theory has proven its effectiveness in numerous scientific applications, such as artificial intelligence and decision-making (Zadeh and Aliev, 2018). The mathematician Atanassov then developed intuitionistic fuzzy set theory in 1983 (Atanassov, 2012) as an extension of fuzzy sets, considered more accurate in handling information ambiguity. While fuzzy sets express the degree to which an element belongs to a particular set, ranging between 0 and 1, Atanassov added the degree of non-membership resulting from conflicting information, such that their sum is less than or equal to 1. This approach proves more accurate and descriptive of uncertain factual data, especially in cases where membership cannot be precisely determined. Intuitionistic fuzzy sets have proven effective in multiple fields such as decision support systems. However, the sum of the membership degree and the non-membership degree being at most 1 may not be sufficient in cases containing contradictory information (Kozae *et al.*, 2020). In neutrosophic sets, indeterminacy is quantified explicitly, and truth-membership, indeterminacy-membership, and false-membership are independent. This assumption is very important in many situations, such as information fusion when combining data from different sensors.

Neutrosophic sets were introduced by Smarandache in the mid-1990s (Wang *et al.*, 2010) as a powerful general formal framework that generalizes classical sets (Purkert and Ilgauds, 2013), fuzzy sets (Zadeh et al., 1996; Zadeh and Aliev, 2018), and intuitionistic fuzzy sets (Atanassov, 2012). Researchers in economics, sociology, medical science, and many other fields deal daily with the complexities of modeling uncertain data (Smarandache and Pramanik, 2016). Classical methods are not always successful because the uncertainty appearing in these domains may be of various types. Neutrosophic sets are characterized by a membership function (μ), an indeterminacy function (w), and a non-membership function (v). This theory is very important in many application areas (Salama and Alblowi, 2012; Hanafy *et al.*, 2012; Al-Omeri and Smarandache, 2016; Wang *et al.*, 2005), since indeterminacy is explicitly quantified and the three membership functions are independent. The concept of similarity is a basic concept in human cognition. Similarity plays an essential role in taxonomy, recognition, case-based reasoning, and many other fields. In the case of set theory-based similarity measures, it is observed that explicit transitivity is a much stronger condition that must be imposed on the similarity measure. In most applications, such as image databases, fuzzy, intuitionistic fuzzy, and neutrosophic logic-based measures seem more appropriate than their explicit counterparts. Even in the case of fuzzy set theory measures, intuitionistic fuzzy set theory measures, and neutrosophic set theory measures, perceived similarities do not follow the property of transitivity. Other models contain a list of axioms that a reasonable similarity measure must satisfy (Ejegwa, 2020; Liang and Shi, 2003; Ye and Zhang, 2014; Beg and Ashraf, 2009; Wang, 1997; Duan and Li, 2021; Chen, 1997; Ali *et al.*, 2022).

This study presents a mathematical model and a comparative analysis rather than an experiment intended to yield clinical generalizations or statistical findings. The dataset comprising seven patients and collected from a medical laboratory in Zawiya, Libya, serves as a case study to illustrate the operational mechanisms of the mathematical models and highlight the differences between them. By applying a unified similarity measure to real medical data, the study aims to analyze each theory's capacity to represent uncertainty, while the mathematical validity of the general framework remains independent of the sample size. The concept behind this work is to derive a unified mathematical formulation for similarity measures that encompasses fuzzy, intuitionistic fuzzy, and neutrosophic frameworks within a single equation, thereby enabling a direct structural comparison. It demonstrates that the neutrosophic similarity measure reduces to the other two measures when specific parameter constraints are met, thereby confirming its generality. Furthermore, the study presents a systematic comparative analysis of the mathematical structures themselves, independent of any domain-specific assumptions. Unlike previous studies that applied each framework in isolation, our work offers a unified perspective that elucidates the theoretical relationships existing among these uncertainty models (Asmizal and Wan Daud, 2025). Liver enzymes are vital indicators of the health and function of the body's heaviest organ.

The liver plays a crucial role in storing vitamins and detoxifying the body. When liver cells malfunction, the levels of these enzymes rise. Therefore, it is important to understand the significance of these enzymes, the causes and symptoms of their elevation, and how to manage them. Elevated liver enzymes sometimes indicate a liver problem, identified through blood tests. These enzymes include ALT, AST, and bilirubin (total bilirubin) (Center, 2007; Kalas *et al.*, 2021).

2. Mathematical Foundations of Uncertainty Theories

Classical sets were the cornerstone of set theory for a long time, and many practical applications relied on them. However, they are very limited in dealing with ambiguous, inaccurate, contradictory, or incomplete data. For this reason, many modern theories have emerged that are more flexible in dealing with these problems, making them a strong foundation in modern applications. In our study, we will focus on three theories: fuzzy set, intuitionistic fuzzy set, and neutrosophic set theories.

2.1 Fuzzy Set Theory

I- Basic Concepts

Definition 2.1.1 (Zadeh *et al.*, 1996)

Let X be non-empty set, a fuzzy set (FS) A in X is a set of ordered pairs $A = \{ (x, \mu_A(x)) : x \in X \}$, where, $\mu_A(x) : X \longrightarrow [0,1]$ is called the membership function, and for each $x \in X$, the value of $\mu_A(x)$ is called the degree of membership of x *in* A. We can write A as the following representation

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), (x_3, \mu_A(x_3)), \dots \dots (x_n, \mu_A(x_n))\}.$$

Example 2.1.1

Let $X = \{1,2,3,4,5,6. \dots .\}$ are the positive real number, membership function for $A = \{1,2,3\}$ in this universal is given as $\mu_A(1) = 0, \mu_A(2) = 0.5, \mu_A(3) = 1$.

Definition 2.1.2 (Zadeh *et al.*, 1996)

Let$x \in X$, then x is called:

1. not included in the fuzzy set if $\mu_A(x) = 0$.
2. Partials include if $0 < \mu_A(x) < 1$.
3. Full included if $\mu_A(x) = 1$.

Definition 2.1.3

A fuzzy set is empty if and only if its membership function is zero for all $x \in X, (i.e\ \mu(x) = 0_F)$.

Definition 2.1.4

The fuzzy universal set U of X is defined $\mu(x) = 1_F$ for all $x \in X$.

Example 2.1.2

Let $X = \{x_1, x_2, x_3\}$ be a nonempty set, $A = \{(x_1, 0), (x_2, 0), (x_3, 0)\}, B = \{(x_1, 0.3), (x_2, 0.1), (x_3, 0.9)\}, C = \{(x_1, 1), (x_2, 1), (x_3, 1)\}$, since $\mu_A(x_1) = 0, \mu_A(x_2) = 0, \mu_A(x_3) = 0$. Then A is fuzzy empty. In addition, $\mu_B(x_1) = 0.3, \mu_B(x_2) = 0.1, \mu_B(x_3) = 0.9$. Then B is fuzzy set. Also, $\mu_C(x_1) = 1, \mu_C(x_2) = 1, \mu_C(x_3) = 1$. Then C fuzzy set universal.

Definition 2.1.5 (Zadeh *et al.*, 1996)

Let A and B be two fuzzy sets; we say that A is a subset of B denoted $A \subseteq B$ if and only if

$\mu_A(x) \leq \mu_B(x)$ for all $x \in X$. Furthermore, A is a proper subset of B denoted $A \subset B$ if and only if $\mu_A(x) < \mu_B(x)$ for all $x \in X$.

Proposition 2.1.1

Let X be a non-empty set and A ⊆ X be a fuzzy set. Then the following hold:

1. $0_F \subseteq A$
2. $0_F \subseteq 0_F$
3. $A \subseteq 1_F$
4. $1_F \subseteq 1_F$

Definition 2.1.6

Let A and Type equation here. be two fuzzy sets, we say that A *and* B are equal denoted$A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$.

II- Fundamental Operations on Fuzzy Sets

The fundamental operations on fuzzy sets include union, intersection, complement and difference, as outlined below (Jamil, 2018; Lee, 2005).

1. Complement law: $\mu_{A^c}(x) = 1 - \mu_A(x), \forall\, x \in X$.
2. Intersection law: $\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)], \forall\, x \in X$.
3. Union law: $\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)], \forall\, x \in X$.
4. Difference law: $\mu_{A-B}(x) = \mu_{A\cap B^c}(x) = \min[\mu_A(x), \mu_{B^c}(x)], \forall x \in X$.

Example 2.1.3

Let $X = \{a, b, c\}$ be a universal set and $A = \{(a, 0.1), (b, 0.4), (c, 0.2)\}, B = \{(a, 0.2), (b, 0.6), (c, 0.3)\}$

$\mu_{A^c}(x) = \{0.9, 0.6, 0.8\}, then\ A^c = \{(a, 0.9), (b, 0.6), (c, 0.8)\}$.

$\mu_{B^c}(x) = \{0.8, 0.4, 0.7\}, then B^c = \{(a, 0.8), (b, 0.4), (c, 0.7)\}$.

$\mu_{A\cup B}(x) = \{0.2, 0.6, 0.3\}, then\ A \cup B = \{(a, 0.2), (b, 0.6), (c, 0.3)\}$.

$\mu_{A\cap B}(x) = \{0.1, 0.4, 0.2\}, then\ A \cap B = \{(a, 0.1), (b, 0.4), (c, 0.2)\}$.

$\mu_{A-B}(x) = \{0.1, 0.4, 0.2\}, then\ A - B = \{(a, 0.1), (b, 0.4), (c, 0.2)\}$.

III- Properties of Fuzzy Sets

Fuzzy sets exhibit properties analogous to those of classical (crisp) sets, which are restricted to binary membership values {0, 1} (Buckley and Eslami, 2002). The following properties hold for any fuzzy sets A, B, and C:

1. Commutativity laws: $A \cup B = B \cup A$, $A \cap B = B \cap A$.
2. Associativity laws: $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$.
3. Distributivity laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. The idempotency laws: $A \cup A = A$, $A \cap A = A$.
5. Identity laws: $A \cup \emptyset = A$, $A \cap X = A$, $A \cup X = X$.
6. Involution law: $(A^c)^c = A$.
7. Demorgan's laws: $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$.

Remark 2.1.1.

In contrast to classical sets, fuzzy sets do not satisfy the laws of contradiction and excluded middle: $A \cap A^c \neq 0_F$, $A \cup A^c \neq 1_F$.

Example 2.1.4

According to Example2.1.3, we have $\mu_{A\cap A^c}(x) = \{0.1, 0.4, 0.2\}, thus\ A \cap A^c = \{(a, 0.1), (b, 0.4), (c, 0.2)\} \neq 0_F$.

$\mu_{A\cup A^c}(x) = \{0.9, 0.6, 0.8\}, thus\ A \cup A^c = \{(a, 0.9), (b, 0.6), (c, 0.8)\} \neq 1_F$.

2.2- Intuitionistic Fuzzy Set Theory

Fuzzy sets are useful for dealing with ambiguity, but they are not ideal for applications requiring high precision. Therefore, intuitionistic fuzzy sets are more powerful and flexible than traditional fuzzy sets because they add a third dimension: the hesitation margin.

I- Basic Concepts

Definition 2.2.1 (Atanassov, 2012)

Let X be a non-empty set. An intuitionistic fuzzy set (IFS) $A \subseteq X$ is an object of the form$A = \{(x, \mu_A(x), \upsilon_A(x): x \in X\}$, where the functions $\mu_A(x) and\ \upsilon_A(x): X \longrightarrow [0,1]$ define, respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set$A$, and for every element $x \in X$, $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$.

Definition 2.2.2 (Atanassov, 2012)

Let $A \in X$ be an IFS. Then $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A *where* $\pi_A(x) : X \longrightarrow [0,1]$ and $0 \leq \pi_A(x) \leq 1$ for every$x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether $x \in X$ belongs to IFS A or not.

Example 2.2.1

Let A be an intuitionistic fuzzy set with $\mu_A(x) = 0.3\ and\ \upsilon_A(x) = 0.1$ then $\pi_A(x) = 1 - 0.3 - 0.1 = 0.6$. This means that the degree that x belongs to IFS A is 0.3, the degree that x does not belong to IFS A is 0.1 and the degree of indeterminacy 0.6.

Definition 2.2.3

An intuitionistic fuzzy set is empty denoted 0_{IF} if and only if its membership function$\mu(x) = 0_{IF}$and non-membership function $\nu(x) = 1_{IF}$ for all $x \in X$.

Definition 2.2.4

The intuitionistic fuzzy universal set U of X denoted1_{IF} is defined membership function$\mu(x) = 1_{IF}$and non-membership function $\nu(x) = 0_{IF}$ for all $x \in X$.

Example 2.2.2

Let $X = \{a, b, c\}$ be a non-empty set and$A = \{(a, 0,1), (b, 0,1), (c, 0,1)\}, B = \{(a, 1,0), (b, 1,0), (c, 1,0)\}$, then $\mu_A(x) = 0,\ \upsilon_A(x) = 1$. So A is empty intuitionistic fuzzy set. $\mu_B(x) = 1,\ V_B(x) = 0$. So$B$ *is* universal intuitionistic fuzzy set.

Definition 2.2.5 (Atanassov, 2012; Kozae *et al.*, 2020)

Let X be a non-empty set. For two IFSs A and B, we say $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\upsilon_A(x) \geq \upsilon_B(x), \forall\, x \in X$.

2.2.1 Proposition

Let X be a non-empty set. For any intuitionistic fuzzy set $A \subseteq X$, the following hold:

1. $0_{IF} \subseteq A$.
2. $0_{IF} \subseteq 0_{IF}$.
3. $A \subseteq 1_{IF}$.
4. $1_{IF} \subseteq 1_{IF}$.

Definition 2.2.6

Let X be a non-empty set, A and B are equal if and only if $A \subseteq B\ and\ B \subseteq A$.

II- Fundamental Operations on Intuitionistic Fuzzy Sets:

For every two IFSs A and B we define the following relations and operations as follows

1- **The complement law:** $A^c = \{x, \upsilon_A(x), \mu_A(x); x \in X\}$

2- **The intersection law:** $A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\upsilon_A(x), \upsilon_B(x)) : x \in X\}$

3- **The union law:** $A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\upsilon_A(x), \upsilon_B(x)) : x \in X\}$

4- **The difference law:**

5- $A - B = \{(x, \min(\mu_A(x), \upsilon_B(x)), \max(\upsilon_A(x), \mu_B(x)) : x \in X\}$

Example 2.2.3

Let $X = \{1,2\}$ be a non-empty set. $A = \{(1,0.5,0.4), (2,0.1,0.3)\}, B = \{(1,0.2,0.1), (2,0.5,0.3)\}$

$A^c = \{(1,0.4,0.5), (2,0.3,0.1)\}$, $A \cap B = \{(1,0.2,0.4), (2,0.1,0.3)\}$, $A \cup B = \{(1,0.5,0.1), (2,0.5,0.3)\}$.

$A - B = \{(1,0.1,0.4), (2,0.1,0.5)\}$.

III- Properties of Intuitionistic Fuzzy Sets (Ejegwa *et al.*, 2014)

Let A, B and C be IFSs in X. The following properties hold:

1- **Commutative laws**: $A \cup B = B \cup A$, $A \cap B = B \cap A$.

2- **Associative laws:** $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$.

3- **Distribution law:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4- **Idempotent Laws:** $A \cup A = A$, $A \cap A = A$.

5- **Complement law:** $(A^c)^c = A$.

6- **De Morgan's laws:** $(A \cup B)^c = A^c \cap B^c, (A \cap B)^c = A^c \cup B^c$.

Remark 2.2.1

In contrast to classical sets, intuitionistic fuzzy sets do not satisfy the laws of contradiction and excluded middle: $A \cap A^c \neq 0_{IF}$, $A \cup A^c \neq 1_{IF}$.

Example 2.2.4

According to Example 2.2.3, we have $A \cap A^c = \{(1,0.4,0.5), (2,0.1,0.3)\} \neq 0_{IF}$,

$$A \cup A^c = \{(1,0.5,0.4), (2,0.3,0.1)\} \neq 1_{IF}.$$

2.3- Neutrosophic Set Theory

Despite the strength and flexibility of intuitionistic fuzzy sets in modeling uncertainty, there is difficulty in dealing with complex information. Therefore, models that are more capable of representing real situations that contain uncertainty or unclear information have emerged, more accurately than fuzzy sets and intuitionistic fuzzy sets, namely neutrosophic sets.

I- Basic Concepts

Definition 2.3.1 (Wang *et al.*, 2010; Smarandache and Pramanik, 2016)

Let X be a nonempty fixed set. A neutrosophic set A (NS for short) is an object having the from $= \{(x, \mu_A(x), \omega_A(x), \upsilon_A(x)), x \in X\}$, where $\mu_A(x) : X \longrightarrow [0,1], \omega_A(x) : X \longrightarrow [0,1]$ *and* $\upsilon_A(x) : X \longrightarrow [0,1]$ represent the degrees of membership , indeterminacy *and* non-membership, respectively of each element $x \in X$ to the set A. These values satisfy the following conditions: $0 \leq \mu_A(x) + \omega_A(x) + \upsilon_A(x) \leq 3$.

Remark 2.3.1 (Smarandache and Pramanik, 2016)

In neutrosophic philosophy, the three functions mentioned above, can be expressed by truth, indeterminacy, and falsity functions as follows: $\mu_A(x) = T_A$, $\omega_A(x) = I_A$, $\upsilon_A(x) = F_A$.

For each $x \in X$, the values T_A, I_A and F_A represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following conditions $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3.2 (Wang *et al.*, 2010)

The neutrosophic empty set denoted 0_N can be defined in four equivalent ways:

1. $0_N = \{(x, 0,0,1), x \in X\}$
2. $0_N = \{(x, 0,1,1), x \in X\}$
3. $0_N = \{(x, 0,1,0), x \in X\}$
4. $0_N = \{(x, 0,0,0), x \in X\}$

Definition 2.3.3 (Wang *et al.*, 2010)

The neutrosophic universe set and denoted 1_N can be defined in four equivalent ways:

1. $1_N = \{(x, 1,0,0), x \in X\}$
2. $1_N = \{(x, 1,0,1), x \in X\}$
3. $1_N = \{(x, 1,1,0), x \in X\}$
4. $1_N = \{(x, 1,1,1), x \in X\}$

Definition 2.3.4 (Wang *et al.*, 2010)

Let X be a non-empty set. Let A and B be neutrosophic sets in X, where

$A = (x, \mu_A(x), \omega_A(x), \upsilon_A(x))$, $B = (x, \mu_B(x), \omega_B(x), \upsilon_B(x))$.

The subset relation $A \subseteq B$ may be defined in one of the following ways:

1. $A \subseteq B = \mu_A(x) \subseteq \mu_B(x), \omega_A(x) \subseteq \omega_B(x), \upsilon_A(x) \supseteq \upsilon_B(x), \forall x \in X$
2. $A \subseteq B = \mu_A(x) \subseteq \mu_B(x), \omega_A(x) \supseteq \omega_B(x), \upsilon_A(x) \supseteq \upsilon_B(x), \forall x \in X$

Proposition 2.3.1

Let X be a non-empty set. For any neutrosophic set $A \subseteq X$, the following hold:

1. $0_N \subseteq A$
2. $0_N \subseteq 0_N$
3. $A \subseteq 1_N$
4. $1_N \subseteq 1_N$

Definition 2.3.5

Two neutrosophic sets A and B are equal, if and only if $A \subseteq B$ *and* $B \subseteq A$.

II- Fundamental Operations of Neutrosophic Sets

The following operations are defined on neutrosophic sets (Salama and Alblowi, 2012).

1. **The complement:** Let $A = (x, \mu_A(x), \omega_A(x), \upsilon_A(x))$ be a neutrosophic set on X. The complement of A (A^c *or* $C(A)$, *for short*) maybe defined in three ways:

i. $A^c = \{(x, 1 - \mu_A(x), 1 - \omega_A(x), 1 - \upsilon_A(x)), x \in X\}$.

ii. $A^c = \{(x, \upsilon_A(x), 1 - \omega_A(x), \mu_A(x)), x \in X\}$.

iii. $A^c = \{(x, \upsilon_A(x), \omega_A(x), \mu_A(x)), x \in X\}$.

2. The intersection and the union

- Let A and B be neutrosophic sets in X. Their intersection and union may be defined as follows:
- $A \cap B$:

i. $A \cap B = min(\mu_A(x), \mu_B(x)), min(\omega_A(x), \omega_B(x)), max(\upsilon_A(x), \upsilon_B(x)), \forall x \in X$

ii. $A \cap B = \min(\mu_A(x), \mu_B(x)), \max(\omega_A(x), \omega_B(x)), \max(\upsilon_A(x), \upsilon_B(x)), \forall x \in X$

- $A \cup B$:

i. $A \cup B = \max(\mu_A(x), \mu_B(x)), \max(\omega_A(x), \omega_B(x)), \min(\upsilon_A(x), \upsilon_B(x)), \forall x \in X$

ii. $A \cup B = \max(\mu_A(x), \mu_B(x)), \min(\omega_A(x), \omega_B(x)), \min(\upsilon_A(x), \upsilon_B(x)), \forall x \in X$

Proposition 2.3.2 (Al-Omeri and Smarandache, 2016)

Let $\{A_j : j \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

- $\cap A_j$ maybe defined as:

i. $\cap A_j = (\wedge \mu_{Aj}(x), \wedge \omega_{Aj}(x), \vee \upsilon_{Aj}(x), \forall j \in J)$.

ii. $\cap A_j = (\wedge \mu_{Aj}(x), \vee \omega_{Aj}(x), \vee \upsilon_{Aj}, \forall j \in J)$.

- $\cup A_j$ maybe defined as:

i. $\cup A_j = (\vee \mu_{Aj}(x), \vee \omega_{Aj}(x), \wedge \upsilon_{Aj}, \forall j \in J)$.

ii. $\cup A_j = (\vee \mu_{Aj}(x), \wedge \omega_{Aj}(x), \wedge \upsilon_{Aj}, \forall j \in J)$.

3. The difference

Let A and B be neutrosophic sets in X. The difference $A - B = A \cap B^c$ is given by:

i. $A - B = min(\mu_A(x), \mu_{B^c}(x)), min(\omega_A(x), \omega_{B^c}(x)), max(\upsilon_A(x), \upsilon_{B^c}(x))$

ii. $A - B = min(\mu_A(x), \mu_{B^c}(x)), max(\omega_A(x), \omega_{B^c}(x)), max(\upsilon_A(x), \upsilon_{B^c}(x))$.

Example 2.3.1

Let $X = \{a, b, c\}$ and $A = \{(a, 0.3, 0.4, 0.1), (b, 0.2, 0.1, 0.8), (c, 0.1, 0.5, 0.8)\}$, $B = \{(a, 0.5, 0.2, 0.6), (b, 0.8, 0.7, 0.3), (c, 0.2, 0.7, 0.3)\}$. Then $A^c = \{(a, 0.1, 0.4, 0.3), (b, 0.8, 0.1, 0.2), (c, 0.8, 0.5, 0.1)\}$.

$B^c = \{(a, 0.6, 0.2, 0.5), (b, 0.3, 0.7, 0.8), (c, 0.3, 0.7, 0.2)\}$.

$A \cap B = \{(a, 0.3, 0.2, 0.6), (b, 0.2, 0.1, 0.8), (c, 0.1, 0.5, 0.8)\}$.

$A \cup B = \{(a, 0.5, 0.4, 0.1), (b, 0.8, 0.7, 0.3), (c, 0.2, 0.7, 0.3)\}$.

$A - B = \{(a, 0.1, 0.4, 0.3), (b, 0.2, 0.1, 0.8), (c, 0.1, 0.5, 0.8)\}$.

III- Properties on Neutrosophic Sets

We will give some properties of set-theoretic operations defined on neutrosophic sets (Wang et al., 2005).

1. Commutativity laws: $A \cup B = B \cup A$, $A \cap B = B \cap A$.
2. Associativity laws: $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$.
3. Distributivity laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
4. Idempotent Laws: $A \cup A = A$, $A \cap A = A$.
5. $(A^c)^c = A$.
6. Demorgan's laws: $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$.

Remark 2.3.2

In contrast to classical sets, neutrosophic sets do not satisfy the laws of contradiction and excluded middle: $A \cap A^c \neq 0_N$, $A \cup A^c \neq 1_N$

Example 2.3.2

According to Example 2.3.1

$A \cap A^c = \{(a, 0.1, 0.4, 0.3), (b, 0.2, 0.1, 0.8), (c, 0.1, 0.5, 0.8)\} \neq 0_N$.

$A \cup A^c = \{(a, 0.3, 0.4, 0.1), (b, 0.8, 0.1, 0.2), (c, 0.8, 0.5, 0.1)\} \neq 1_N$.

We present in the next table a summary of a simplified comparison among the three aforementioned uncertainty theories.

Table 1

Comparison between Uncertainty Theories

Feature	fuzzy sets	Intuitionistic fuzzy sets	Neutrosophic sets
Introduced by	Lotfi Zadeh (1965)	Atanasov (1983)	Smarandache (1995)
Membership	$\mu \in [0,1]$	$\mu, \nu \in [0,1]$	$\mu, \omega, \upsilon \in]0^-, 1^+[$
Uncertainty handling	Partial certainty	Partial certainty+ hesitation	Full uncertainty, contradiction, and indeterminacy
Sum constraint	$\mu \leq 1$	$\mu + \nu \leq 1$	$\mu + \omega + \nu \leq 3$
Application	Control systems, AI, decision-making	Pattern recognition, medical diagnosis	Complex systems, Pattern recognition, medical diagnosis

3- Similarity Measures in Uncertainty Frameworks

Similarity measures between sets are fundamental tools in various scientific applications, particularly in information retrieval, artificial intelligence, and decision-support systems. The significance of these measures lies in their ability to clarify the scientific differences between the three theories (Fuzzy, Intuitionistic Fuzzy, and Neutrosophic) by measuring the proximity of information within each framework. While fuzzy sets rely on a similarity scale to measure the degree of convergence in partial belonging, intuitionistic fuzzy sets extend this scale to measure the similarity of incomplete or contradictory information, taking into account both belonging and non-belonging. neutrosophic sets, on the other hand, provide a more general and realistic scale based on three independent components (truth, ambiguity, and falsity), enhancing their ability to

assess data similarity in most situations, especially those where contradiction, uncertainty, or information gaps are clearly evident.

Similarity measures between two sets A and B are characterized by the following properties:

1. $0 \leq S(A,B) \leq 1$.

2. $S(A,B) = 1\ iff\ A = B$.

3. $S(A,B) = S(B,A)$.

4. If $A \subseteq B \subseteq C$ then $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$

3.1-Fuzzy Similarity Measure

The similarity measure in fuzzy sets is a mathematical tool used to determine the degree of convergence between two sets, yielding a value within the range [0, 1]. This measure is primarily based on the "distance" between sets (such as the Hamming distance), where similarity is viewed as the dual concept of distance: the smaller the distance, the greater the similarity. Numerous similarity measures for fuzzy sets have been proposed in the literature (Beg and Ashraf, 2009). For brevity, we do not present details of the different models of similarity generated. We shall recall only the similarity measures for fuzzy sets that are formulated in the literature (Beg and Ashraf, 2009). In our research, we will rely on a similarity measure analogous to Gregson's model.

Definition 3.1.1 (Beg & Ashraf, 2009)

The Hamming distance is given by the following formula

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}|A(x_i) - B(x_i)| \qquad (1)$$

The similarity measure then is given by the following formula:

$$S_F(A,B) = 1 - d_H(A,B) \qquad (2)$$

3.2- Intuitionistic Fuzzy Similarity Measure

The similarity measure here represents a fundamental tool for measuring the difference between intuitionistic fuzzy sets while considering additional dimensions. The formulation of similarity in this framework relies on three essential parameters for each element: the degree of membership, the degree of non-membership, and the hesitation margin (indeterminacy), making it more precise in evaluating incomplete information. Many types of measures possessing different properties have been proposed in the literature, such as distance and similarity measures (Duan & Li, 2021). We will use the Hausdorff distance to construct a new distance for intuitionistic fuzzy sets because of its simplicity in calculation. We rely on the following formula:

Definition 3.2.1 (Duan & Li, 2021)

Let $A = \{(x, \mu_A(x), \nu_A(x), \pi_A(x))\}, B = \{(x, \mu_B(x), \nu_B(x), \pi_B(x))\}$ be two intuitionistic fuzzy sets, Then the distance between A, B is:

$$H_3(A,B) = \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\}$$

The similarity between A, B is:

$$S_{IF}(A,B) = 1 - H_3(A,B) \qquad (4)$$

3.3-Neutrosophic Similarity Measure

The neutrosophic similarity measure provides the most comprehensive framework, dealing with three completely independent components: truth, indeterminacy, and falsity. Its strength lies in its ability to measure similarity even in cases of contradiction and absolute ambiguity, relying on various neutrosophic distance measures that enhance the efficiency of complex data analysis.

There are many similarity measures for neutrosophic sets. In our study, we will rely on the following Hausdorff distance-based similarity measure (Ye & Zhang, 2014).

Definition 3.3.1 (Ye & Zhang, 2014)

Let $A = \{(x, \mu_A(x), \omega_A(x), \nu_A(x))\}, B = \{(x, \mu_B(x), \omega_B(x), \nu_B(x))\}$ be two neutrosophic sets.

Then the distance between A, B is:

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x) - \mu_B(x)|, |\omega_A(x) - \omega_B(x)|, |\nu_A(x) - \nu_B(x)|\}.$$

Where $d_H(A,B)$ denotes the extended Hausdorff distance.

The similarity measure for neutrosophic sets as follows:

$$S_N(A,B) = 1 - d_H(A,B) \qquad (6)$$

3.4- Unified Similarity Model

The three similarity measures can be formulated within a unified and comprehensive mathematical framework, where the neutrosophic measure is considered the most generalized model. By imposing specific constraints on the parameters of indeterminacy and falsity, this measure naturally reduces to intuitionistic fuzzy or fuzzy similarity measures, allowing them all to be integrated into a single mathematical formula capable of handling various levels of uncertainty.

I- General Measure

The three measures can be integrated into a single, unified mathematical framework. Since Neutrosophic Logic is a generalization of both Fuzzy and Intuitionistic Fuzzy logics, the Neutrosophic Similarity Measure serves as the Universal Formula.

Definition 3.4.1

The normalized distance between any two sets A and B, denoted $d_H(A,B)$ and defined as

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x) - \mu_B(x)|, |\omega_A(x) - \omega_B(x)|, |\nu_A(x) - \nu_B(x)|\} \qquad (7)$$

The unified similarity measure S (A, B), for any two sets A and B, can be defined as

$$S(A,B) = 1 - d_H(A,B) \qquad (8)$$

II-Derivation of Sub-Measures

To Intuitionistic Fuzzy: By assuming the constraint that $\pi_A(x)$ is dependent on $\mu_A(x)$ and $\nu_A(x)$ (where $\mu_A(x) + \pi_A(x) + \nu_A(x)$=1), the formula reduces to the Intuitionistic Fuzzy similarity measure. (3)

To Fuzzy: By setting $\omega_A(x)$= 0 and $\nu_A(x) = 1 - \mu_A(x)$, the formula collapses into the classical Fuzzy similarity measure based on a single membership degree.

This unification demonstrates that the neutrosophic approach is a comprehensive paradigm capable of modeling all levels of uncertainty within a single mathematical expression.

4- Application of Similarity Measures in the Diagnosis of Liver Enzyme Levels

Now, we present a practical application of the three similarity measures to real medical data related to liver enzymes (ALT, AST and B.T). The importance of this application lies in the ability of neutrosophic logic to handle ambiguous medical cases that fall within the realm of "indeterminacy". Studying the average weights of patient laboratory indicators—without focusing on any single

one in isolation—allows for a more accurate comparison with reference cases than traditional estimation methods. The transformation functions were selected based on predefined mathematical membership functions established according to the clinical reference ranges for aspartate aminotransferase (AST), alanine aminotransferase (ALT), and bilirubin; however, these can be replaced by any domain-specific transformation function. The primary contribution lies in applying the same transformation across all frameworks to ensure a fair comparison.

4.1- Overview: Liver Enzyme Disorders

The clinical significance of liver enzymes is a fundamental medical indicator relied upon by physicians and specialists for the diagnosis of various liver diseases. This section focuses on the most important of these enzymes, specifically alanine aminotransferase (ALT) and aspartate aminotransferase (AST), which are considered key biomarkers of liver health. The discussion addresses the normal physiological ranges of each enzyme and the pathological implications of elevated levels, which often indicate liver cell damage (Kalas *et al.*, 2021). This medical background provides the necessary context for understanding the "uncertainty and ambiguity that the mathematical models presented in this paper seek to address within the context of medical diagnosis.

Liver enzyme analysis is one of the most important tests for assessing liver function. Liver enzymes are defined as proteins primarily produced by liver cells that perform multiple functions in the body, such as food digestion and detoxification. Among the most important of these enzymes are:

1. Aspartate aminotransferase (AST)
2. Alanine aminotransferase (ALT)
3. Direct bilirubin (B.T)

The Reference Model for Liver Enzymes (Center, 2007)

AST (SGOT) normal rate $(0-35)\quad U/L$

ALT (SGPT) normal rate $(0-45)\quad U/L$

Direct bilirubin (B.T) normal rate $(0.0-1.2)\quad mg/dl.$

These liver enzymes are present in the blood in limited quantities, and elevated levels, especially AST and ALT, usually indicate leakage from inflamed liver cells.

Causes of Elevated Liver Enzymes (Center, 2007)

Elevated liver enzymes often indicate inflammation or damage to liver cells and other organs, and may be caused by: Hepatitis, Viral hepatitis, Alcohol consumption.

4.2- Practical Application: Comparative Analysis of Patient Cases

This section presents a detailed application of the similarity measures discussed previously on a sample of seven patients. By utilizing real laboratory data for these individuals, the study compares how each of the three theories, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic, calculates the degree of similarity between the patients' results and the reference (healthy) benchmarks.

The liver enzymes dataset in Table 2 includes clinical measurements for patients$(p_1, p_2, \dots . p_7)$, and the symptoms represent Liver enzymes S= {AST, ALT, Bilirubin}.

Table 2

Clinical Patients Values

Patient	AST	ALT	B. T
p_1	29	39	0.74
p_2	23	18	0.8
p_3	75	95	2.3
p_4	120	130	1.7
p_5	195	175	3.5
p_6	220	225	3.25
p_7	350	320	4

The data includes physical measurements and test results. The reference model for liver disease patients is compared with patient P using different uncertainty theories**.**

These data illustrate the variation in enzyme levels, which serve as primary indicators for determining the extent of hepatic cell damage. These values represent 'Crisp Data,' which makes it challenging to reach a definitive diagnostic decision in borderline cases p_3, p_4, p_5. Consequently, this necessitates their transformation into logical values (Fuzzy, Intuitionistic, and Neutrosophic) in the subsequent stages of the research to apply similarity measures with higher precision.

To identify the similarities between patients and derive a clear medical explanation based on the similarity of their symptoms, we follow these steps for each theory:

Step1: Extract new patient data based on the functions of each theory individually.

Step2: Calculate the weighted average of all enzymes.

Step 3: Determine the distance and similarity.

Step4: Extract the medical interpretation of similarity tables when applied to each theory.

First: In Fuzzy Sets

Step 1-Transforming Values

The transformation functions rely on the standard min-max normalization technique, mapping values from their original range to the interval [0, 1]. For a specific enzyme (X) with a normal range $[L, U]$ and a maximum pathological value$U$, the membership function is defined as $\mu(x) = \max\{0, min\left(1, \frac{x-L}{U-L}\right)\}$. This continuous, monotonic function preserves the relative ordering of the original data. Although the use of different monotonic transformations may yield varying absolute similarity values, the relative ranking of patients and the comparative behavior of the three frameworks remain consistent in qualitative terms, a finding supported by recent studies on similarity measures (Sarkar & Ghosh, 2024). We will convert each numerical value for patients in Table 2 to a fuzzy value ranging between 0 and 1. We follow the following procedures.

1- $min - max$ Boundary is a popular technique.

2- Feature scaling that converts the numerical values of features into a designated range of [0,1].

3- This transformation is done by subtracting the feature's minimum value and dividing by the range, which is the difference between the maximum and minimum values.

4- The substitution in the formula used to calculate the minimum and maximum for each enzyme as follows:

$$\mu(x) = \begin{cases} 0 & x \le L \\ \frac{x-L}{U-L} & L < x < U \\ 1 & x \ge U \end{cases}.$$ Where $L = X_{min}$is the feature's minimum value, $U = X_{max}$is its highest value, and x is the feature's original value.

i- Aspartate Aaminotransferase (AST) $\mu(x) =$

$$\begin{cases} 0 & x \le 35 \\ \frac{x-35}{200-35} & 35 < x < 200 \\ 1 & x \ge 200 \end{cases}$$

$\mu(P_1) = 29 \le 35\ then\ \mu(P_1) = 0\ ,\ \mu(P_2) = 23 \le 35\ then\ \mu(P_2) = 0\ ,\ \mu(P_3) = 0.24$

$\mu(P_4) = 0.52, \mu(P_5) = 0.97, \mu(P_6) = 220 > 200\ then\ \mu(P_6) = 1$

$\mu(P_7) = 350 > 200\ then\ \mu(P_7) = 1$

ii- Alanine Aminotransferase (ALT) $\mu(x) = \begin{cases} 0 & x \leq 45 \\ \frac{x-45}{200-45} & 45 < x < 200 \\ 1 & x \geq 200 \end{cases}$

$\mu(P_1) = 39 \leq 45$ *then* $\mu(P_1) = 0$, $\mu(P_2) = 18 \leq 45$ *then* $\mu(P_2) = 0, \mu(P_3) = 0.32$

$\mu(P_4) = 0.55$, $\mu(P_5) = 0.84$, $\mu(P_6) = 225 > 200$ *then* $\mu(P_6) = 1, \mu(P_7) = 1.$

iii- Direct Bilirubin (B.T) $\mu(x) = \begin{cases} 0 & x \leq 1.2 \\ \frac{x-1.2}{3-1.2} & 1.2 < x < 3 \\ 1 & x \geq 3 \end{cases}$

$\mu(P_1) = 0.74 \leq 1.2$ *then* $\mu(P_1) = 0$, $\mu(P_2) = 0.8 \leq 1.2$ *then* $\mu(P_2) = 0$, $\mu(P_3) = 0.61$

$\mu(P_4) = 0.28, \mu(P_5) = 3.5 > 3$ *then* $\mu(P_5) = 1, \mu(P_6) = 1, \mu(P_7) = 1.$

The fuzzy values for the three enzymes of the seven patients can be summarized in the following table.

Table 3

Fuzzy Patients Values

Patient	μ_{AST}	μ_{ALT}	$\mu_{B.T}$
p_1	0	0	0
p_2	0	0	0
p_3	0.24	0.32	0.61
p_4	0.52	0.55	0.28
p_5	0.97	0.84	1
p_6	1	1	1
p_7	1	1	1

Table 3 converts the original laboratory values presented in Table 2 into fuzzy grades ($\mu \in [0,1]$) to indicate severity:

0 indicates to Normal, [0-1] indicates to Mild to moderate elevation (borderline cases),1 indicates Severe elevation Fuzzy data highlights the most affected enzymes and provides a faster, clearer, and more precise assessment, especially for borderline patients. For example, patient P3 shows moderate B.T elevation (μ = 0.61) despite mild AST, ALT increase, which is less obvious in the raw data. In contrast, patient p_4 shows a slight increase in AST, ALT despite a decrease in B.T.

Therefore, Fuzzy representation improves the interpretation of borderline cases.

Step 2- Weighted Average:

To combine the three symptoms, we use the weighted average $\mu_{total} = \frac{\mu_{AST}+\mu_{ALT}+\mu_{B.T}}{3}$

$\mu_{p_1} = 0$, $\mu_{p_2} = 0$, $\mu_{p_3} = 0.31$, $\mu_{p_4} = 0.45$, $\mu_{p_5} = 0.94$, $\mu_{p_6} = 1$, $\mu_{p_7} = 1.$

By calculating the weighted average, comparison with reference model for liver enzymes and compared to traditional estimation methods multiple individual symptoms (AST, ALT, and B.T) are fused into a single score. This provides a holistic view of the patient's health rather than looking at isolated enzymes. For example, in Table 3, patientp_3, by looking at the first and second enzymes, the situation seems reassuring, while the third enzyme indicates an alarming situation, putting the doctor in a dilemma about making the appropriate decision. However, through the weighted average, which showed that $\mu_{p_3} = 0.31$, a periodic follow-up was requested from the patient. On the other hand, patient p_4 had relatively high values for the first and second enzymes and low for the third. Based on the weighted average, the value was $\mu_{p_4} = 0.45$, higher than that ofp_3, which led the doctor to request additional tests and more precise analyses. It highlights the most affected enzymes, allowing for a faster and more precise clinical evaluation.

Step 3 Applying Fuzzy Similarity between Patients

From Definition 3.1.1, we get $d(P_1, P_1) = 0, d(P_1, P_2) = 0, d(P_1, P_3) = 0.39$, $d(P_1, P_4) = 0.45$, $d(P_1, P_5) = 0.94$, $d(P_1, P_6) = 1, d(P_1, P_7) = 1, d(P_2, P_2) = 0$, $d(P_2, P_3) = 0.39$

$d(P_2, P_4) = 0.45, d(P_2, P_5) = 0.94, d(P_2, P_6) = 1, d(P_2, P_7) = 1, d(P_3, P_3) = 0, d(P_3, P_4) = 0.28$ $d(P_3, P_5) = 0.55, d(P_3, P_6) = 0.61, d(P_3, P_7) = 0.61, d(P_4, P_4) = 0, d(P_4, P_5) = 0.49$, $d(P_4, P_6) = 0.55, d(P_4, P_7) = 0.55$, $d(P_5, P_5) = 0, d(P_5, P_6) = 0.06$, $d(P_5, P_7) = 0.06, d(P_6, P_6) = 0, d(P_6, P_7) = 0, d(P_7, P_7) = 0.$

Now, we obtain the similarity between patients as shown in the following table.

Table 4

Fuzzy Similarity

p_i/p_j	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_1	1	1	0.61	0.55	0.06	0	0
p_2	1	1	0.61	0.55	0.06	0	0
p_3	0.61	0.61	1	0.72	0.45	0.39	0.39
p_4	0.55	0.55	0.72	1	0.51	0.45	0.45
p_5	0.06	0.06	0.45	0.51	1	0.94	0.94
p_6	0	0	0.39	0.45	0.94	1	1
p_7	0	0	0.39	0.45	0.94	1	1

Step 4 Fuzzy Medical Interpretation

Based on results provided in Table 4, for the patient similarity matrix and relying on liver biomarkers (AST, ALT, and bilirubin), seven patients are classified into three distinct clinical groups. These clusters reflect varying degrees of hepatocyte integrity and bile duct function.

Group1: Normal Liver function(p_1, p_2):

It is clear that patients (p_1, p_2) are identical (S = 1). This indicates that they have an identical biochemical signature, thus confirming that they represent the control group or patients with normal liver function. While their complete lack of similarity to p_6, p_7group (S = 0), confirms a complete absence of pathological indicators, found in severe cases.

Group 2: Severe Hepatic Impairment (p_5, p_6, p_7):
This cluster shows high internal cohesion. p_6 and p_7 are identical (S = 1.00), and both share a high affinity with p_5 ($S = 0.94$).

The high similarity coefficients in this group indicate a con-

sistent pattern of markedly elevated enzymes and hyperbilirubinemia. This pattern confirms advanced and severe liver cell damage (e.g., acute hepatitis, advanced cirrhosis, or obstructive jaundice). The sharp contrast with the first group clearly underscores the sensitivity of the fuzzy measure in identifying apparent pathologies.

Group 3: Borderline Clinical Zone (p_3, p_4):
these patients occupy a "fuzzy" transitional state. (P_3, P_4) show moderate similarity to the healthy cluster (S: $0.55 - 0.61$) and lower similarity to the severe group(S: $0.39 - 0.51$).
This group represents subclinical or borderline hepatic dysfunction. The intermediate values suggest early-stage liver disease, fatty liver (NAFLD), or recovering hepatic tissue. Unlike the polar groups, these patients exhibit "ambiguous" membership, necessitating further diagnostic stratification, such as imaging (Ultrasound/Fibro Scan) or longitudinal monitoring of liver function tests (LFTs). The fuzzy measure in identifying overt pathology.

Therefore, the matrix effectively differentiates between physiological stability and pathological derangement. The high similarity within the P_5, P_7 group facilitates a rapid diagnostic grouping for severe cases, while the intermediate coefficients for P_3 and P_4 highlight the necessity for a differentiated clinical approach to prevent progression toward overt liver failure.

Second: In Intuitionistic Fuzzy Theory

Step 1-Transforming Values

To obtain the values of the functions in intuitionistic fuzzy theory for each enzyme, we use this method: We establish the μ values as calculated in Table 3 using the aforementioned linear functions and their proximity to the reference values. After that, we determine the ν value so that the sum of the two values ranges between 0 and 1 for each enzyme, according to the clinical context of the diagnosis, based on the following medical principles:

Non-membership values ν were assumed to reflect the physician's confidence in the patient's well-being based on each enzyme reading. This assumption relied on proximity to or distance from the normal range, with the ν value reflecting the physician's confidence that the enzyme reading does not indicate liver dysfunction. However, it gradually decreases as the deviation from the normal range increases. The sensitivity of each enzyme was also considered, as the ν values reflect the degree to which each enzyme is sensitive as an indicator of liver disease. The higher the enzyme's sensitivity, the greater its effect on lowering the value. Meanwhile, the values of π were chosen to represent the physician's margin of confusion or degree of diagnostic uncertainty, which, as it increases, warrants further investigation.

Table 5

Intuitionistic Fuzzy Patients Values

P	μ_{AST}	ν_{AST}	π_{AST}	μ_{ALT}	ν_{ALT}	π_{ALT}	$\mu_{B.T}$	$\nu_{B.T}$	$\pi_{B.T}$
p_1	0	1	0	0	1	0	0	1	0
p_2	0	1	0	0	1	0	0	1	0
p_3	0.24	0.53	0.23	0.32	0.44	0.24	0.61	0.01	0.38
p_4	0.52	0.03	0.45	0.55	0.04	0.41	0.28	0.71	0.01
p_5	0.97	0.01	0.02	0.84	0.03	0.13	1	0	0
p_6	1	0	0	1	0	0	1	0	0
p_7	1	0	0	1	0	0	1	0	0

Table 6

Intuitionistic Fuzzy Similarity

p_i/p_j	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_1	1	1	0.33	0.26	0.01	0	0
p_2	1	1	0.33	0.26	0.01	0	0
p_3	0.33	0.33	1	0.58	0.44	0.39	0.39
p_4	0.26	0.26	0.58	1	0.48	0.45	0.45
p_5	0.01	0.01	0.44	0.48	1	0.73	0.73
p_6	0	0	0.39	0.45	0.73	1	1
p_7	0	0	0.39	0.45	0.73	1	1

Step 4 Intuitionistic Fuzzy Medical Interpretation

Based on results in Table 6, of the patient similarity matrix and depending on liver biomarkers the seven patients can be divided into three distinct clinical groups. These groups reflect varying degrees of hepatocyte integrity and bile duct function.

Group 1: Normal Liver Profile(p_1, p_2):

Degree of similarity between them 1 $(S = 1)$ patients (P_1, P_2) show complete similarity, indicative biochemical signature. They are likely to represent:

Control group (healthy individuals), patients with normal liver function, weak similarity to other patients (P_6, P_7) is not sufficient to classify them into other groups

Group 2: patients with severe liver dysfunction with biliary involvement (P_5, P_6, P_7)

The degree of patients' similarity ranges from 0.73 *to* 1 (high to complete similarity). These patients show high similarity, suggesting:

- Advanced liver damage with significantly elevated liver enzymes.
- Significant biliary dysfunction, which may include:
  - Cholestasis (cholestatic cholestasis).

- Cirrhosis
- Primary sclerosing cholangitis.
- Possible jaundice or elevated bilirubin levels.

The complete similarity between (P_6, P_7) suggests an almost identical clinical presentation, possibly in an advanced stage of the same disease.

Group 3: patients with moderate hepatic impairment (P_3, P_4).

The degree of similarity between them: 0.47 similarity middle. The moderate similarity between these two patients reflects the presence:

- Moderate liver dysfunction, which may be the result of chronic hepatitis or the beginning of liver cell damage.
- Moderate elevation in liver enzymes (AST/ALT), with possible slight effect on bile ducts.
- Conditions such as chronic hepatitis or non-advanced fatty liver may be present.

There is partial similarity with group 2, which may indicate a transitional phase toward worsening

This classification reflects the gradation in the severity of liver disease, from normal function to systemic dysfunction, which is consistent with the normal progression in chronic and acute liver diseases.

Third: In Neutrosophic Set Theory

Step 1 Transforming Values

We will consider the membership and non-membership values for each enzyme as previously calculated in Table 5, we would determine the value of ω, such that their sum ranges between 0 and 3. This is because, in some pathological cases, the physician may be certain that the patient has a disease (high μ), but has significant doubts (high ω) due to some conflicting indicators, while simultaneously having some confidence that it may not be a disease (moderate ν). Therefore, this complex situation is represented by three independent values.

Table 7

Neutrosophic Patients Values

P	μ_{AST}	ω_{AST}	ν_{AST}	μ_{ALT}	ω_{ALT}	ν_{ALT}	$\mu_{B.T}$	$\omega_{B.T}$	$\nu_{B.T}$
p_1	0	0	1	0	0	1	0	0	1
p_2	0	0	1	0	0	1	0	0	1
p_3	0.24	0.70	0.53	0.32	0.99	0.44	0.61	0.01	0.01
p_4	0.52	1	0.03	0.55	0.01	0.04	0.28	0.90	0.71
p_5	0.97	0.01	0.01	0.84	0.46	0.03	1	0.99	0
p_6	1	1	0	1	1	0	1	1	0
p_7	1	1	0	1	1	0	1	1	0

In Table 7, using a neutrosophic framework, patients' data are represented through three independent components $(\mu_A(x), \omega_A(x), \upsilon_A(x))$ allowing more expressive description of medical uncertainty. Instead of relying solely on fuzzy or intuitionistic thresholds, neutrosophic logic captures ambiguous and borderline variations in liver function indicators (AST, ALT, B. T). The neutrosophic representation highlights the most affected biomarkers and provides a clearer and faster assessment of patient conditions, particularly in cases where traditional methods fail to distinguish mild or uncertain abnormalities. For example, although patient P_3 shows only slight increases in AST and ALT in raw measurements, the neutrosophic values assign higher truth-membership to abnormality while preserving a degree of indeterminacy, thus reflecting hidden clinical risk.

Therefore, consequently, neutrosophic sets offer a flexible and comprehensive framework for modeling complex medical cases, enabling the simultaneous treatment of certainty, uncertainty, and contradiction as independent quantities, and supporting more reliable decision-making compared with fuzzy or intuitionistic approaches.

Step 2-Weighted Average

In addition to the weights of the two previous functions that were calculated (the membership and non- membership functions), we will calculate the weight of the weighted indeterminate function in the same way. We have $\mu_{p_1} = 0,\ \mu_{p_2} = 0,\ \mu_{p_3} = 0.31,\ \mu_{p_4} = 0.45,\ \mu_{p_5} = 0.94,\ \mu_{p_6} = 1,\ \mu_{p_7} = 1$.Also, $\nu_{p_1} = 1, \nu_{p_2} = 1,\ \nu_{p_3} = 0.33\ , \nu_{p_4} = 0.26,\ \nu_{p_5} = 0.01, \nu_{p_6} = 0,\ \nu_{p_7} = 0$. And $\omega_{total} = \frac{\omega_{AST}+\omega_{ALT}+\omega_{B.T}}{3}$

$\omega_{p_1} = 0,\ \omega_{p_2} = 0,\ \omega_{p_3} = 0.57\ \ , \omega_{p_4} = 0.64,\ \omega_{p_5} = 0.49,\ \omega_{p_6} = 1,\ \omega_{p_7} = 1.$

In neutrosophic sets, however belonging is represented by three independent components: the degree of membership μ, the degree of indeterminacy ω and the degree of non-membership ν. Each of these components takes a value in the interval [0,1], and their sum does not have to be equal to one rather it is governed by the relationship.

$$0 \le \mu(x) + \omega(x) + \nu(x) \le 3$$

This representation allows greater flexibility in modeling the ambiguities, uncertainties and contradictions found in data, for example, a patient's condition can be represented by a triad (where represents the degree of certainty of the disease's presence, the degree of certainty of its absence, and the ν degree of uncertainty or lack of information. If a patient's condition is given by these values $\mu_{p_3} = 0.31\ , \omega_{p_3} = 0.57, \nu_{p_3} = 0.33$ this means there is a relatively low degree of certainty about the existence of the disease, while a high degree of ambiguity 0.57 prevails, indicating the need for further investigations or information. Similarly, other cases can be represented as a patient with a high degree of disease certainty$(\mu = 1, \omega = 0, \nu = 0)$, implying a definitive diagnosis. symptom incorporation (AST, ALT, B. T) in neutrosophic multiple symptoms can be incorporated into a three- part vector that expresses overall health status, while retaining the ambiguity and uncertainty dimension as an independent value, giving a richer picture than intuitive clustering.

Step 3- Applying Neutrosophic Similarity between Patients

From Definition 5.3.1, we have: $d_H(P_1, P_2) = 0, S_N(P_1, P_2) = 1$

$d_H(P_1, P_3) = 0.89,\ S_N(P_1, P_3) = 0.11. H(P_1, P_4) = 0.95,\ H(P_1, P_5) = 0.99,\ H(P_1, P_6) = 1, H(P_1, P_7) = 1, H(P_2, P_2) = 0,\ H(P_2, P_3) = 0.89\ , H(P_2, P_4) = 0.95, H(P_2, P_5) = 0.99, H(P_2, P_6) = 1,\quad H(P_2, P_7) = 1,\ H(P_3, P_3) = 0,\quad H(P_3, P_4) = 0.82,\quad H(P_3, P_5) = 0.74,\quad H(P_3, P_6) = 0.81, H(P_3, P_7) = 0.81,\ H(P_4, P_4) = 0, H(P_4, P_5) = 0.72, H(P_4, P_6) = 0.73, H(P_4, P_7) = 0.73,\ H(P_5, P_5) = 0, H(P_5, P_6) = 0.51,\ H(P_5, P_7) = 0.51, H(P_6, P_6) = 0, H(P_6, P_7) = 0, H(P_7, P_7) = 0$

Now we use the similarity measure, we get the following table:

Table 8

Neutrosophic Similarity

p_i/p_j	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_1	1	1	0.11	0.05	0.01	0	0
p_2	1	1	0.11	0.05	0.01	0	0
p_3	0.11	0.11	1	0.18	0.26	0.19	0.19
p_4	0.05	0.05	0.18	1	0.28	0.27	0.27
p_5	0.01	0.01	0.26	0.28	1	0.49	0.49
p_6	0	0	0.19	0.27	0.49	1	1
p_7	0	0	0.19	0.27	0.49	1	1

Step 4- Neutrosophic Medical Interpretation

The seven patients can be classified into three clinical groups based on the neutrosophic similarity table and neutrosophic liver biomarkers. These groups reflect varying degrees of hepatocyte integrity and biliary tract function, taking into account the uncertainty in clinical assessment:

Group 1: Normal Liver Profile(p_1, p_2):

These two patients show a very high neutrosophic similarity value (1), with very low similarity to the other groups. This indicates a normal liver condition with a limited degree of diagnostic uncertainty, thus confirming to the clinician the absence of liver damage or biliary tract dysfunction.

Group 2: Advanced liver dysfunction/ high similarity(P_5, P_6, P_7).

The neutrosophic similarity score among the three patients was (1), indicating that these patients had complete similarity among themselves, a high degree of similarity to the intermediate group, and no similarity to the normal group. This, in turn, reflects an advanced stage of chronic liver disease, with impaired liver and biliary function, thus demonstrating a high degree of certainty and therefore a clear clinical diagnosis.

Group 3: transitional phase / intermediate hepatic dysfunction(P_3, P_4).

This group lies in the middle zone of neutrosophic similarity, with similarity values ranging from moderate to relatively high (between 0.18 *and* 0.28), with low similarity to the normal group and relatively high similarity to the advanced group. This reflects a transitional or intermediate phase of hepatic dysfunction, such as chronic fatty liver or the onset of systemic influence on liver function, with a marked degree of clinical uncertainty.

5-General Medical Interpretation and Decision-Making

5.1- Classification of Patients with Normal Values

The three models showed high similarity among patients within the normal range. Differences between the frameworks were minor. The diagnostic decision was stable across all theories.

Therefore, in clear cases with biochemical stability, all models are effective.

5.2- For Patients with Moderately Elevated Enzymes (transitional conditions):

Fuzzy model yielded relatively high intermediate similarity scores, suggesting a possible inclusion within a general disease category.

Intuitionistic model showed decreased similarity due to the emergence of a frequency margin.

Neutrosophic model revealed a significant increase in the indeterminacy score ω, with μ and ν remaining in equilibrium.

Therefore, the neutrosophic model did not immediately classify the condition as pathological but indicated a transitional condition requiring further laboratory investigation.

5.3- Patients with Conflicting Results

In cases where: A markedly elevated ALT, with AST near normal and B.T. within acceptable limits

Fuzzy: The condition was classified as intermediate.

Intuitionistic: Showed variability but its source was not identified.

Neutrosophic: Showed simultaneous elevations in μ and ν, with a moderate ω value

Therefore, this pattern reflects genuine diagnostic variability, possibly due to an early stage of the disease or variability in patient response.

5.4- Patients with Severe Cases:

In cases of significant increases across all indicators:

All models showed high similarity within the disease category. However, the neutrosophic model showed a marked decrease in ω (uncertainty), reflecting the clarity of the medical decision. Therefore, the more severe the disease, the lower the degree of uncertainty.

Numerically, the following were observed:

The similarity distance between borderline and severe cases increased in the neutrosophic model.

The probability of misclassification decreased.

The sensitivity for detecting transitional cases has improved. Therefore, the neutrosophic model supports the decision to continue, re-examine, or intervene early instead of a potentially hasty, definitive decision.

The validity of our comparative results can be theoretically assessed based on the consistency of the similarity measures with the axioms defined in Section 3. All three-similarity measures satisfy the properties of reflexivity, symmetry, and transitivity, while the monotonicity of the transformation functions ensures internal consistency. Our findings align with a recent study (Mustapha *et al.*, 2024), which indicated improved classification using neutrosophic

sets in medical diagnosis and demonstrated the superiority of neutrosophic similarity in handling borderline cases. However, unlike those empirical studies, our research provides a mathematical explanation for this superiority by proving that the neutrosophic measure generalizes the other measures and by demonstrating how the independence of the indeterminacy component (ω) allows for more precise distinctions—a perspective supported by recent theoretical analyses (Sarkar & Ghosh, 2024; Mustapha *et al.*, 2024).

6-Conclusion:

The results showed that the evolution of the mathematical structure from a single representation of belonging to a two-part, and then to a three-part, independent representation of truth, indeterminacy, and falsity, clearly improved the model's discriminatory power. While fuzzy sets provided acceptable handling of clear cases, intuitionistic fuzzy sets improved the characterization of incompleteness, and the neutrosophic framework achieved the highest level of diagnostic ability and differentiation between normal, transitional, and severe cases, especially when there were conflicting laboratory indicators. Therefore, the results of this study confirm that the neutrosophic model represents the most comprehensive and appropriate framework for complex environments, especially medical environments, in which ambiguity and uncertainty are highly apparent. It is worth noting that the study was designed to compare mathematical behaviors rather than to evaluate prediction accuracy through statistical validation methods; the appropriate validation criteria for this type of study are mathematical consistency and structural comparison—principles emphasized in foundational literature on neutrosophic sets and studies regarding similarity measures.

Future work: applying the method to larger public datasets, incorporating statistical validation, and optimizing transformation processes using machine learning.

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